

Meson Supermultiplet Decay Constants

N.R. Jones and R. Delbourgo

Physics Department, University of Tasmania,
GPO Box 252C, Hobart, Tas. 7001, Australia.

Abstract

We use a covariant supermultiplet theory to determine the primary coupling constant associated with several types of two-body meson decay. Despite the diverse range of decays considered the primary coupling constant is surprisingly uniform. We envisage the extension of the techniques to heavy quark cases, including as preliminary examples the calculation of the D^{*+} and D^{*0} total decay widths with results 57.7 ± 1.5 KeV and 42.5 ± 2.6 KeV respectively, as well as some predictions about D^* and B^* radiative decays.

1 Introduction

In these heady days of heavy quark effective theories (HQET) we feel it is appropriate to review a supermultiplet scheme developed in the mid-1960's which has great similarities to the HQET in respect of the heavy quark and the accompanying 'brown muck'. In doing so we hope to test the covariant supermultiplet theory for light as well as heavy degrees of freedom, assessing the extent of symmetry breaking and how it manifests itself. We may then apply our techniques to heavy quark examples in future work with considerable confidence.

To determine the matrix elements of currents between hadrons requires knowledge of the hadronic wave function in terms of the quark and gluon constituents. A full relativistic treatment of such constituents is impossible because of the infinite degrees of freedom associated with the quarks and gluons. However, the success of the non-relativistic quark model spawned several workers [1, 2, 3] to construct relativistic spinor fields describing pointlike mesons incorporating the correct spin, parity, flavour and colour degrees of freedom. Despite later evidence that the mesons were not pointlike objects, these group theoretical approaches can be shown to be equivalent to the non-relativistic weak binding limit [4].

The basis of our work is a relativistic meson supermultiplet field [5]. The wave-function describing the meson is dynamically equivalent to a system of two quarks, both of which are on-shell and moving at the same velocity. This differs little from the heavy quark picture which assumes that a meson with a quark much heavier than its light antiparticle partner will have the heavy component almost on-shell and moving at the same velocity as the meson because the light 'brown muck' [6] must move with the same velocity. This assumption, along with the hope that weak interactions will not affect the motion of the heavy quark at small recoil, led to the now popular heavy quark symmetry and decoupling (see [7, 8] for a review).

The reasons for using the supermultiplet scheme are several. Firstly, the scheme automatically incorporates the Zweig rules and duality diagrams, so one can easily determine the Zweig allowed strong decays. At the same time it incorporates isospin and field mixing factors so that one can readily normalise the coupling constant of various decays; indeed this makes supermultiplet theory very predictive as one need only know a single coupling constant to predict widths of many seemingly unrelated processes. Secondly, radiative decay modes may be examined by combining the supermultiplet scheme with the vector meson dominance model. This permits the

theory to make some quite accurate predictions about photon mediated decays such as $\omega \rightarrow \pi^+\pi^-$. Thirdly, excited mesonic states may be constructed in terms of the supermultiplet field [9] so we are able to further broaden its applications. Finally, the scheme is easily extended to include the c [10] and b quark flavoured mesons so that we may venture into the heavy quark arena with little modification.

The recent ACCMOR [11] and CLEO Collaborations [12] have renewed interest in the D^* decays due to two major findings. The D^{*+} total width was measured with an upper bound of 131 KeV (significantly lower than the 1992 upper bound of 1.1 MeV), and the $D^{*+} \rightarrow D^+\gamma$ branching fraction appears significantly smaller than earlier measurements. Both these findings are consistent with constituent quark model [13] and HQET predictions [14]. We provide similar calculations within the supermultiplet framework and reproduce these findings with considerably far ease.

2 Supermultiplet Field

Derivation of the wave function [1] proceeds by applying a relativistic boost to the rest frame spinor $\phi_a^b(\hat{p})$, odd under parity. This leads to a relativistic spinor $\phi_A^B(p)$, satisfying Bargmann–Wigner equations, namely

$$\phi_A^B(p) = \phi_{a\alpha}^{b\beta}(p) = (\not{p} + m)[\gamma^\mu \phi_{\mu a}^b - \gamma_5 \phi_{5a}^{b\beta}]/2m \quad (1)$$

where a, b are flavour indices, α, β are spin indices and ϕ_5 corresponds to the pseudoscalar nonet and ϕ_μ to the vector nonet (with $p^\mu \phi_\mu = 0$). One can show [4] that such a relativistic spinor is equivalent to describing the meson as a quark-antiquark pair both of which are moving at the same velocity as the meson and are therefore both on-shell.

We use the simplest effective interaction Lagrangian

$$\mathcal{L}_{int} = G \Phi_A^B(p_1) [\Phi_B^C(p_2) \Phi_C^A(p_3) + (p_2 \leftrightarrow p_3)] \quad (2)$$

as proposed by [1] to describe the three point coupling between the mesons involved in two-body decays. G is a normalization factor and we note that it has the dimensions of mass. Such an interaction Lagrangian corresponds to a duality diagram [15, 16] as shown in Figure 1 with three mesons meeting at a vertex (p_i incoming). Here flavour labels have been included to show how the flavour is automatically conserved, flavour being carried by the line. Contravariant spinor indices correspond to odd

parity as they represent the antiquark, while covariant indices have even parity since they represent the quark flavour.

Upon substitution of the supermultiplet field (1) into the interaction Lagrangian (2), and using the Dirac trace algebra along with the momentum conditions

$$p_1 + p_2 + p_3 = 0, \quad p_i^2 = m_i^2, \quad \phi(i) = \phi(p_i)$$

we reduce the interaction Lagrangian to

$$\mathcal{L}_{int} = g_{VPP} (p_2 - p_3)^\mu < \phi_\mu(1) [\phi_5(2), \phi_5(3)] > \quad (3)$$

$$+ g_{VVP} \epsilon^{\mu\nu\kappa\lambda} p_{1\kappa} p_{2\lambda} < \phi_\mu(1) \{ \phi_\nu(2), \phi_5(3) \} > \quad (4)$$

$$+ g_{VVV} [(p_2 - p_3)^\mu g^{\nu\sigma} m_1 + (p_3 - p_1)^\nu g^{\sigma\mu} m_2 + (p_1 - p_2)^\sigma g^{\mu\nu} m_3 \\ + 2(p_2 - p_3)^\mu (p_3 - p_1)^\nu (p_1 - p_2)^\sigma / (m_1 + m_2 + m_3)] \\ < \phi_\mu(1) [\phi_\nu(2), \phi_\sigma(3)] > \quad (5)$$

where $<>$ stands for a trace over the internal symmetry indices, corresponding to a joining of quark lines in a duality diagram. For instance, such a trace for flavour indices would expand as

$$\phi_{\mu a}^b(1) (\phi_{5b}^c(2) \phi_{5c}^a(3) - \phi_{5b}^c(3) \phi_{5c}^a(2))$$

for the vector–pseudoscalar–pseudoscalar (VPP) vertex. The interaction Lagrangian contains three distinct coupling constants as at this stage we are alert to the possibility that symmetry breaking may affect each piece differently, that is we have introduced three separate constants depending on the type of decay. However, full supermultiplet symmetry would mean the constants are related in the following way:

$$2g_{VPP} = m_1 g_{VVP} = m_1 g_{VVV} \quad (6)$$

One should also note that the three pseudoscalar meson vertex is not present in the interaction Lagrangian, as expected by parity conservation. (This follows automatically in the supermultiplet scheme because the trace over three ϕ_5 fields disappears from Equation 2.)

For the moment we only consider strong meson decay so that tree-level calculations in perturbation theory given by (3, 4, 5) suffice. Also we apply the general formula for a two-body decay,

$$\Gamma_{1 \rightarrow 2,3} = \frac{\lambda^{1/2}(m_1^2, m_2^2, m_3^2)}{16\pi m_1^3 (2s_1 + 1)} \sum_{\text{spins}} |\mathcal{L}_{int}|^2 \quad (7)$$

where s_1 is the spin of the parent meson and $\lambda(m_1^2, m_2^2, m_3^2) = m_1^4 + m_2^4 + m_3^4 - 2m_1^2m_2^2 - 2m_1^2m_3^2 - 2m_2^2m_3^2$.

Upon substitution of the interaction Lagrangian in the form of Equations (3,4,5) into the decay rate formula (7), we derive the following widths for the various decays:

$$\Gamma_{V \rightarrow PP} = \lambda^{3/2}(m_1^2, m_2^2, m_3^2) g_{VPP}^2 / 48\pi m_1^5 \quad (8)$$

$$\Gamma_{V \rightarrow VP} = \lambda^{3/2}(m_1^2, m_2^2, m_3^2) g_{VVP}^2 / 96\pi m_1^3 \quad (9)$$

$$\Gamma_{V \rightarrow VV} = \lambda^{3/2}(m_1^2, m_2^2, m_3^2) g_{VVV}^2 \mathcal{Y}(m_1, m_2, m_3) / 192\pi m_1^5 m_2^2 m_3^2, \quad (10)$$

where

$$\begin{aligned} \mathcal{Y}(m_1, m_2, m_3) = & 9 \left(\sum_{1 \leq i < j \leq 3} (m_i + m_j)^2 (m_i - m_j)^4 - \sum_{i=1}^3 m_i^6 \right) + \\ & \prod_{i=1}^3 m_i \left(98 \sum_{i=1}^3 m_i^3 - 16 \sum_{1 \leq i < j \leq 3} (m_i + m_j)^3 \right) + 142 \prod_{i=1}^3 m_i^2. \end{aligned}$$

We now have an adequate formalism for describing various strong interaction decays amongst ground state mesons. To extend the applications of the supermultiplet theory we invoke the ideas of the vector meson dominance model in order to account for various electromagnetic interactions of our mesons. To do so we make the usual assumption that the coupling between a vector meson flavour singlet and a photon is of the form

$$g_{V\gamma}(k^2 = 0) = em_V^2 / g_{VPP}, \quad (11)$$

as shown in Figure 2. When extrapolating away from $k^2 = 0$ we expect the coupling to decrease and as such have denoted the coupling by em_V^2 / g'_{VPP} to allow for such change. That is, we anticipate $g'_{VPP}(k^2)$ will vary with k^2 due to intermediate virtual particle contributions and its value at $k = 0$ equals g_{VPP} in Equation 3.

The vector meson dominance model, used in conjunction with our decay rate formulae (8,9,10) give the following rates for the various processes:

$$\Gamma_{V \rightarrow \bar{l}l} = (m_V^2 - 4m_l^2)^{1/2} (1 - m_l^2/m_V^2)^{1/2} (e^2/g'_{VPP})^2 / 12\pi \quad (12)$$

$$\Gamma_{V \rightarrow P\gamma} = (m_V^2 - m_P^2)^3 (eg_{VVP}/g'_{VPP})^2 / 96\pi m_V^3 \quad (13)$$

$$\Gamma_{P \rightarrow V\gamma} = (m_P^2 - m_V^2)^3 (eg_{VVP}/g'_{VPP})^2 / 32\pi m_P^3 \quad (14)$$

$$\Gamma_{P \rightarrow \gamma\gamma} = m_P^3 (e^2 g_{VVP} / g'_{VPP} g'_{VPP})^2 / 64\pi \quad (15)$$

thereby greatly extending the original scope of the supermultiplet scheme. In going from our purely strong interaction decay rates to the radiative ones, we have used the

gauge invariance of our interaction Lagrangian and simply substituted a mass of zero for those vectors connecting with the photon. However, the three vector interaction (5) is only gauge invariant for the case $m_2 = m_3$ so strictly we should only apply it to radiative examples for which the virtual vector meson satisfies this condition. Unfortunately, since the photon only couples to flavour singlet states the condition $m_2 = m_3$ also implies the daughter vector mesons are identical. Due to the F-type coupling between daughter states in the interaction Lagrangian (5) such decay widths will automatically go to zero. It is for this reason we have not included a $V \rightarrow V\gamma$ term above, despite experimental evidence for such (eg. $\Gamma_{\phi \rightarrow \rho\gamma}/\Gamma_{\phi \rightarrow \text{all}} < 2\%$; although our zero width prediction does not conflict with this). We now go on to apply the formalism to the ground state mesons.

3 Supermultiplet Method

In the standard way we take the pseudoscalar nonet as:

$$\phi_{5a}^b \xrightarrow{0^-} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} \end{pmatrix} \quad (16)$$

where

$$\eta_8 = \cos \theta_P \eta - \sin \theta_P \eta' \quad (17)$$

$$\eta_0 = \sin \theta_P \eta + \cos \theta_P \eta' \quad (18)$$

as defined in [17] and θ_P is the pseudoscalar mixing angle. The vector nonet is similarly given by

$$\phi_{\mu a}^b \xrightarrow{1^-} \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_0}{\sqrt{3}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_0}{\sqrt{3}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\frac{2\omega_8}{\sqrt{6}} + \frac{\omega_0}{\sqrt{3}} \end{pmatrix} \quad (19)$$

where

$$\omega_8 = \cos \theta_V \phi - \sin \theta_V \omega \quad (20)$$

$$\omega_0 = \sin \theta_V \phi + \cos \theta_V \omega \quad (21)$$

and θ_V is the vector mixing angle as determined using the Gell-Mann–Okubo (GMO) mass relation. We have chosen to go into some detail as our formalism is not the

common one adopted by recent publications [18, 19]. Conversely,

$$\phi = \cos \theta_V \omega_8 + \sin \theta_V \omega_0 \quad (22)$$

and the ω field as

$$\omega = -\sin \theta_V \omega_8 + \cos \theta_V \omega_0, \quad (23)$$

where ω_8 and ω_0 masses are determined by the GMO relation [17]. The vector mixing angle is obtained from

$$\tan 2\theta_V = \frac{2((m_\phi^2 - m_8^2)(m_8^2 - m_\omega^2))^{1/2}}{2m_8^2 - m_\phi^2 - m_\omega^2}, \quad (24)$$

where $3m_8^2 = 4m_{K^*}^2 - m_\rho^2$.

From the vector nonet (19) the octet and singlet fields are expressed in terms of the supermultiplet vector as

$$\sqrt{6} \omega_8 = \phi_{\mu 1}^1 + \phi_{\mu 2}^2 - 2\phi_{\mu 3}^3, \quad \sqrt{3} \omega_0 = \phi_{\mu 1}^1 + \phi_{\mu 2}^2 + \phi_{\mu 3}^3.$$

Substituting these into Equation 22 yields

$$\sqrt{6} \phi = (\cos \theta_V + \sqrt{2} \sin \theta_V)(\phi_{\mu 1}^1 + \phi_{\mu 2}^2) + (-2 \cos \theta_V + \sqrt{2} \sin \theta_V)\phi_{\mu 3}^3.$$

In the case of “ideal mixing” $\phi = \phi_{\mu 3}^3$ so that

$$\cos \theta_V + \sqrt{2} \sin \theta_V = 0 \quad \text{or} \quad \tan \theta_V = -1/\sqrt{2}$$

leaving us two options for θ_V ; either $-\pi/2 < \theta_V < 0$ or $\pi/2 < \theta_V < \pi$. The first case implies $\cos \theta_V = \sqrt{2/3}$, $\sin \theta_V = -1/\sqrt{3}$ so that $\phi = -\phi_{\mu 3}^3$ while the second gives the desired result of $\phi = \phi_{\mu 3}^3$. Thus a suitable solution to Equation 24 is in the range $\pi/2 < \theta_V < \pi$. More generally, the solution to (24) is

$$2\theta_V = \tan^{-1} \left(\frac{2((m_\phi^2 - m_8^2)(m_8^2 - m_\omega^2))^{1/2}}{2m_8^2 - m_\phi^2 - m_\omega^2} \right) + n\pi$$

where n is any integer.

The above arguments for the determination of the vector mixing angle can be applied to the pseudoscalar nonet with the substitutions $\omega_8 \rightarrow \eta_8$, $\omega_0 \rightarrow \eta_0$, $\phi \rightarrow \eta$, $\omega \rightarrow \eta'$. Using the condition $\pi/2 < \theta_V < \pi$ and a similarly derived expression for the pseudoscalar angle, $-\pi/2 < \theta_P < \pi/2$, we obtain the equally likely results

$$\begin{aligned} \theta_V &= 129.4^\circ, 140.6^\circ \\ \theta_P &= -10.5^\circ, 10.5^\circ. \end{aligned}$$

The mixing angles we have obtained may seem accurate, but the GMO relation is extremely sensitive to an extra small SU(3) symmetry breaking associated with the $\underline{27}$ representation; a small $\underline{27}$ addition can produce a *major* modification of the angle.

With the correct structure now in place, it is a relatively simple process to test the supermultiplet theory. We wish to calculate the standard coupling constants g_{VPP} and g_{VVP} , examine how similar they are for each process and finally compare the supermultiplet prediction (6) of the relation between them. In practice we take the decay width and particle masses as input [19] and determine the coupling constant associated with the decay via (8,9) and (12–15). The simplicity of the supermultiplet method is that isospin and mixing factors are automatically accounted for. One simply chooses an appropriate decay, determines the flavour indices a, b and c using matrices (16,19) and then use these in the correct part of the interaction Lagrangian (3, 4, or 5) to determine the normalization factors which arise. For example, in the decay $\rho^+ \rightarrow \pi^+ \pi^0$, $a = 1$, $b = 2$, $c = 1, 2$ and upon substitution of the fields into Equation 3 one finds $g_{\rho^+ \pi^+ \pi^0} = \sqrt{2} g_{VPP}$ so that the coupling constant we determine for this decay should be divided by the factor $\sqrt{2}$ to obtain the standard coupling constant g_{VVP} . This procedure is repeated for all appropriate physical decays. Mixing is easily accommodated by using the relations (17,18,20,21) to replace the ideal fields by the real mesons in the interaction Lagrangian.

In radiative decays of the type $V \rightarrow l \bar{l}$ we allow for the coupling of the photon to the quark. Using the following electromagnetic charge projectors

$$Q_a^b = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} \quad (25)$$

we may likewise extract the relevant standard coupling.

Radiative modes such as $V \rightarrow P\gamma$ require some delicacy in normalising the coupling constant. To elicit a clear understanding of the method we include an example of the procedure for the decay $\rho^0 \rightarrow \eta\gamma$. Firstly, we recognise ρ^0 is a combination of $\phi_{\mu 1}^1(1)$ and $\phi_{\mu 2}^2(1)$, so we require terms in the interaction Lagrangian (4) with $a = 1$, $b = 1$ and $a = 2$, $b = 2$. For each of these cases we determine the third flavour index c such that $\phi_{\nu}(2)$ is a flavour singlet that couple to a photon (thus $c = 1, 2$). Substituting these values into formula 4 we pick out the uncharged parts;

$$\mathcal{L}_{int} \propto g_{VVP} [\phi_{\mu 1}^1(1) \{ \phi_{\nu 1}^1(2), \phi_{51}^1(3) \} + \phi_{\mu 2}^2(1) \{ \phi_{\nu 2}^2(2), \phi_{52}^2(3) \}]$$

$$\begin{aligned}
&= g_{VVP} \left[\frac{\rho^0(1)}{\sqrt{2}} \left(2 \left(\frac{\rho^0(2)}{\sqrt{2}} + \frac{\omega_8(2)}{\sqrt{6}} \right) \left(\frac{\eta_8(3)}{\sqrt{6}} + \frac{\eta_0(3)}{\sqrt{3}} \right) \right. \right. \\
&\quad \left. \left. - \frac{\rho^0(1)}{\sqrt{2}} \left(2 \left(\frac{-\rho^0(2)}{\sqrt{2}} + \frac{\omega_8(2)}{\sqrt{6}} \right) \left(\frac{\eta_8(3)}{\sqrt{6}} + \frac{\eta_0(3)}{\sqrt{3}} \right) \right) \right] \\
&= \sqrt{\frac{2}{3}} g_{VVP} \rho^0(1) \rho^0(2) [\eta_8(3) + \sqrt{2}\eta_0(3)] \\
&= \sqrt{\frac{2}{3}} (\cos \theta_P + \sqrt{2} \sin \theta_P) g_{VVP} \rho^0(1) \rho^0(2) \eta(3),
\end{aligned}$$

where in particular we have used relations (17,18) to arrive at the final result. The form shows that the coupling between two ρ^0 mesons and a pseudoscalar η is related to the standard VVP coupling by

$$g_{\rho^0 \rho^0 \eta} = \sqrt{\frac{2}{3}} (\cos \theta_P + \sqrt{2} \sin \theta_P) g_{VVP} \quad (26)$$

The virtual vector meson is immediately identifiable as $\rho^0(2)$, and we must necessarily allow for the coupling between this and the photon. Since $\rho^0 = (u\bar{u} + d\bar{d})/\sqrt{2}$ then $g_{\rho^0 \gamma} = g_{V\gamma}/\sqrt{2}$ which in turn implies $g'_{\rho^0 PP} = \sqrt{2}g'_{VPP}$ from (11). Subsequently, the coupling between a ρ^0 , η and photon is related to our standard couplings by

$$\begin{aligned}
g_{\rho^0 \eta \gamma} &= e g_{\rho^0 \rho^0 \eta} / g'_{\rho^0 PP} \\
&= \frac{e \sqrt{\frac{2}{3}} (\cos \theta_P + \sqrt{2} \sin \theta_P) g_{VVP}}{\sqrt{2} g'_{VPP}} \\
&= \frac{1}{\sqrt{3}} (\cos \theta_P + \sqrt{2} \sin \theta_P) \frac{e g_{VVP}}{g'_{VPP}}
\end{aligned}$$

In other decays, it is possible that the radiative mode may proceed via more than one virtual vector meson. The above method is still used to determine each virtual vector meson contribution and the appropriate linear combination is taken.

4 Results

The results are presented in Tables 1 and 2. For clarity these tables include the SU(3) factors which we have used to normalize the coupling constant.

Table 1 summarises the results of our investigation into the coupling between a vector meson and two pseudoscalar mesons. The first half of Table 1 displays purely strong interaction decays, while the second lists the coupling constant g'_{VPP} obtained from vector meson dominance extrapolation. Two important features are apparent. Firstly, we have found that the coupling is far more regular than previously believed by those persons who deprecate light quark symmetry. Secondly, the form of the symmetry breaking is now very clear. As the mass of the parent vector meson

increases (as we go down each half of the table), so does the coupling, apparently following the simple rule $g_{VPP} \approx 0.154 m_V^{1/2}$ (for m_V in units MeV). Similarly, the mass-shell constants g'_{VPP} follow such a relation, except the constant of proportionality is approximately $(0.136 \pm 0.003)\text{MeV}^{-1/2}$ by a weighted mean method (and an error scale factor of 4; following the Particle Data Group's handling of errors). This result complies with the known scaling law behaviour for f_V as $m_V \rightarrow \infty$ [8]. Thus if the matrix elements of quark currents between a given vector meson and the vacuum state is defined by

$$\langle 0 | \bar{q}_2 \gamma_\mu q_1 | V \rangle = f_V m_V \epsilon_\mu$$

then it is well known [20] that $f_V \propto |\psi(0)| / m_V^{1/2}$ as $m_V \rightarrow \infty$. Translating to our terminology $f_V = e m_V / g'_{VPP}$, we verify this prediction and importantly we find the result is also supported in the light meson sector. Admittedly, the $\omega \rightarrow e^+ e^-$ has a very high g'_{VPP} , but since the width of $\omega \rightarrow \mu^+ \mu^-$ is only known to an upper bound (providing a lower bound estimate of g'_{VPP}) the anomaly remains unsubstantiated.

Table 2 predominantly lists the results from studying radiative decays to obtain estimates of g_{VVP} using vector meson dominance. The first entry in the table is for the decay $\phi \rightarrow \rho\pi$ and leads to a direct determination g_{VVP} , not via a radiative transition. In fact, we use it to test the supermultiplet prediction $2 g_{VPP} = m_1 g_{VVP}$, the results of which are shown in Figure 3. This figure shows the sum $m_1 g_{VVP} - 2 g_{VPP}$ plotted against vector mixing angle θ_V and clearly demonstrates the supermultiplet rule is satisfied at $\theta_V \approx 140.3^\circ$, very close to the accepted value $\theta_V = 140.6^\circ$, and it is for this reason we have used this value in all our calculations.

In the case of radiative decays, where we know the coupling is related to the ratio g_{VVP}/g'_{VPP} , we have used the relationship $g'_{VPP} \approx 0.136 m_V^{1/2}$, which is well supported by the data in Table 1. Importantly, this relation applies to the virtual vector meson so that for decays mediated via the ideal field ω_8 we have to use its mass of approximately 931 MeV. The data shows that once again the coupling is quite regular, but now the symmetry breaking appears to obey a power law relation $g_{VVP} \propto m_1^{-n}$ where $1/2 < n < 3/2$.

5 Predictions

With a clearer understanding of the effects of symmetry breaking on the coupling constant, we may now confidently determine the decay rates for non-Zweig allowed

decays. In particular we study the decays $\omega \rightarrow \pi^+\pi^-$ and $\phi \rightarrow \pi^+\pi^-$, both of which are mediated by a virtual photon coupling between the parent vector meson and a ρ meson (electromagnetic mixing). Thus in the case $\omega \rightarrow \pi^+\pi^-$ we have the overall coupling of

$$g_{\omega\pi\pi} = e^2 m_\rho^2 g_{\rho\pi\pi} / g'_{\omega PP} (m_\omega^2 - m_\rho^2) g'_{\rho PP}$$

and for $\phi \rightarrow \pi^+\pi^-$ we have

$$g_{\phi\pi\pi} = e^2 m_\rho^2 g_{\rho\pi\pi} / g'_{\phi PP} (m_\phi^2 - m_\rho^2) g'_{\rho PP}.$$

Using

$$\begin{aligned} g_{\rho\pi\pi} &= \sqrt{2} (0.1537 \pm 0.002) m_\rho^{1/2} \\ g'_{\rho PP} &= \sqrt{2} (0.136 \pm 0.003) m_\rho^{1/2} \\ g'_{\omega PP} &= \sqrt{6} (0.136 \pm 0.003) m_\omega^{1/2} / \sin \theta_V \\ g'_{\phi PP} &= \sqrt{6} (0.136 \pm 0.003) m_\phi^{1/2} / \cos \theta_V \end{aligned}$$

we predict

$$\begin{aligned} \Gamma_{\omega \rightarrow \pi^+\pi^-} &= (1.66 \pm 0.16) \times 10^{-2} \text{ MeV} \\ \Gamma_{\phi \rightarrow \pi^+\pi^-} &= (5.88 \pm 0.55) \times 10^{-4} \text{ MeV} \end{aligned}$$

which compare favourably with the presently accepted values

$$\begin{aligned} \Gamma_{\omega \rightarrow \pi^+\pi^-} &= (1.86 \pm 0.25) \times 10^{-2} \text{ MeV} \\ \Gamma_{\phi \rightarrow \pi^+\pi^-} &= (3.5 \pm 2.8) \times 10^{-4} \text{ MeV}. \end{aligned}$$

The symmetry breaking effects we have observed also lead to a measurable consequence in the radiative decays of heavy mesons. We begin by re-examining the decays $K^{*\pm} \rightarrow K^\pm \gamma$ and $K^{*0} \rightarrow K^0 \gamma$. Experimentally, the K^* branching fraction is

$$\Gamma_{K^{*0} \rightarrow K^0 \gamma} / \Gamma_{K^{*+} \rightarrow K^+ \gamma} = 2.31 \pm 0.29$$

and allowing for phase space factors this translates into a coupling constant ratio of

$$|g_{K^{*0} K^0 \gamma} / g_{K^{*+} K^+ \gamma}| = 1.514 \pm 0.095,$$

and as such is far from the exact SU(3) ratio of 2. Under the supermultiplet scheme, one can show the decays proceed via two intermediate vector mesons, ρ^0 and ω_8 .

Following the procedure we described for determining the normalisation factors, one finds

$$g_{K^{*+}K^+\gamma} = e \left(\frac{g_{K^{*+}\rho^0 K^+}}{g'_{\rho^0 PP}} + \frac{g_{K^{*+}\omega_8 K^+}}{g'_{\omega_8 PP}} \right) \quad (27)$$

$$g_{K^{*0}K^0\gamma} = e \left(\frac{g_{K^{*0}\rho^0 K^0}}{g'_{\rho^0 PP}} + \frac{g_{K^{*0}\omega_8 K^0}}{g'_{\omega_8 PP}} \right). \quad (28)$$

If one assumes g'_{VPP} is constant then

$$\begin{aligned} g_{K^{*+}K^+\gamma} &= e \left(\frac{1/\sqrt{2}}{\sqrt{2}} + \frac{-1/\sqrt{6}}{\sqrt{6}} \right) (g_{VVP}/g'_{VPP}) \\ &= g_{VP\gamma}/3 \\ g_{K^{*0}K^0\gamma} &= e \left(\frac{-1/\sqrt{2}}{\sqrt{2}} + \frac{-1/\sqrt{6}}{\sqrt{6}} \right) (g_{VVP}/g'_{VPP}) \\ &= -2g_{VP\gamma}/3, \end{aligned}$$

and we arrive at the exact SU(3) prediction. If instead we use a symmetry breaking g'_{VPP} we must substitute $g'_{\rho^0 PP} = \sqrt{2}C m_{\rho^0}^{1/2}$ and $g'_{\omega_8 PP} = \sqrt{6}C m_{\omega_8}^{1/2}$ in Equations (27) and (28). Thus

$$\frac{g_{K^{*0}K^0\gamma}}{g_{K^{*+}K^+\gamma}} = -\frac{m_{\rho^0}^{-1/2} + m_{\omega_8}^{-1/2}/3}{m_{\rho^0}^{-1/2} - m_{\omega_8}^{-1/2}/3} = -1.87$$

and notice the result is independent of C , the constant of proportionality between g'_{VPP} and $m_V^{1/2}$. Although not matching the experimental result, it is an improvement on exact SU(3). Actually, the most satisfactory explanation of the symmetry breaking mechanism comes from [21]. They attribute the deviation from exact SU(3) to the constituent mass difference between the strange and non-strange quarks in the loop of a quark triangle diagram. As $K^* \rightarrow K\gamma$ excite both strange and non-strange quarks, such a difference must be accounted for. With these corrections, the experimental ratio is found to match theoretical estimates very well. We intend to apply the method to heavier meson cases in future work [22].

Let us continue to use the “mass variation principle” of g'_{VPP} in the heavy meson sector. Upon application to the D^* and B^* mesons we obtain

$$\begin{aligned} \frac{g_{D^{*0}D^0\gamma}}{g_{D^{*+}D^+\gamma}} &= \frac{3m_{\rho^0}^{-1/2} + m_{\omega_8}^{-1/2} + 4m_{J/\psi}^{-1/2}}{-3m_{\rho^0}^{-1/2} + m_{\omega_8}^{-1/2} + 4m_{J/\psi}^{-1/2}} \approx -60 \\ \frac{g_{B^{*0}B^0\gamma}}{g_{B^{*+}B^+\gamma}} &= \frac{-3m_{\rho^0}^{-1/2} + m_{\omega_8}^{-1/2} + 2m_v^{-1/2}}{3m_{\rho^0}^{-1/2} + m_{\omega_8}^{-1/2} + 2m_v^{-1/2}} \approx -0.34 \end{aligned} \quad (29)$$

which are significantly different from the exact SU(5) predictions of

$$\begin{aligned} g_{D^{*0}D^0\gamma}/g_{D^{*+}D^+\gamma} &= 4 \\ g_{B^{*0}B^0\gamma}/g_{B^{*+}B^+\gamma} &= 0 \end{aligned}$$

and as such require better experimental data to test the results.

In addition to these relative decay rate predictions, the supermultiplet scheme can be easily adapted to decay width calculations. Scattered amongst Tables 1 and 2 are various constants determined by extending the supermultiplets to include the charm and bottom quark mesons. In particular, using the upper bound of 131 KeV for the D^{*+} decay width [11] we have found $g_{VPP} < 10$. Conversely, we can use our knowledge of the effects of symmetry breaking to predict the VPP coupling constant for D^{*+} . We find

$$g_{VPP}(D^{*+}) \approx (0.1537 \pm 0.002)(2010)^{1/2} = 6.89 \pm 0.09 \quad (30)$$

surprisingly similar to a heavy quark prediction of 7 ± 1 by [23]. We can use the g_{VPP} value for D^{*+} to calculate the total decay width of the $D^{*+} \rightarrow PP$ channels. Using the supermultiplet method we can predict all the possible decays of D^{*+} into two pseudoscalars; however, phase space restricts the processes to $D^{*+} \rightarrow D^0\pi^+$ and $D^{*+} \rightarrow D^+\pi^0$ so that the width must be

$$\begin{aligned} \Gamma_{D^{*+} \rightarrow PP} &= \Gamma_{D^{*+} \rightarrow D^+\pi^0} + \Gamma_{D^{*+} \rightarrow D^0\pi^+} \\ &= g_{VPP}^2 [\lambda^{3/2}(m_{D^{*+}}^2, m_{D^0}^2, m_{\pi^+}^2) + \lambda^{3/2}(m_{D^{*+}}^2, m_{D^+}^2, m_{\pi^0}^2)] / 48\pi m_{D^{*+}}^5 \\ &= 57.7 \pm 1.5 \text{ KeV}. \end{aligned}$$

We compare this with the radiative width $D^{*+} \rightarrow D^+\gamma$ in the following branching fraction:

$$\begin{aligned} \frac{\Gamma_{D^{*+} \rightarrow D^+\gamma}}{\Gamma_{D^{*+} \rightarrow PP}} &= \frac{(g_{VPP\gamma}/g_{VPP})^2(m_{D^{*+}}^2 - m_{D^+}^2)^3/96\pi m_{D^{*+}}^3}{[\lambda^{3/2}(m_{D^{*+}}^2, m_{D^0}^2, m_{\pi^+}^2) + \lambda^{3/2}(m_{D^{*+}}^2, m_{D^+}^2, m_{\pi^0}^2)]/48\pi m_{D^{*+}}^5} \\ &\approx \frac{(e(-3m_{\rho^0}^{-1/2} + m_{\omega_8}^{-1/2} + 4m_{J/\psi}^{-1/2})/(6 \times 0.1361))^2(m_{D^{*+}}^2 - m_{D^+}^2)^3}{\lambda^{3/2}(m_{D^{*+}}^2, m_{D^0}^2, m_{\pi^+}^2) + \lambda^{3/2}(m_{D^{*+}}^2, m_{D^+}^2, m_{\pi^0}^2)} \\ &\approx (9.48 \pm 0.43) \times 10^{-5}, \end{aligned}$$

where in particular we have used the supermultiplet prediction $g_{VVP}/g_{VPP} = 2/m_{D^{*+}}$ and we have confidence in our prediction to this order. This finding implies that the dominant decay modes in D^{*+} decay are the PP channels we derived and as such

approximate well to the full width. Thus the radiative branching fraction for $D^{*+} \rightarrow D^+ \gamma$ is relatively small. This is not inconsistent with recent measurements by CLEO which measured a fraction of $(1.1 \pm 1.4 \pm 1.6)\%$ [12]. However, our prediction does conflict with other theoretical models [13, 24, 25, 26]. Branching fraction calculations for the two PP channels yield

$$\begin{aligned}\Gamma_{D^{*+} \rightarrow D^0 \pi^+} / \Gamma_{D^{*+} \rightarrow \text{all}} &\approx 68.8\% \\ \Gamma_{D^{*+} \rightarrow D^+ \pi^0} / \Gamma_{D^{*+} \rightarrow \text{all}} &\approx 31.2\%\end{aligned}$$

which compare well with other models and the experimentally determined results from CLEO (1992):

$$\begin{aligned}\Gamma_{D^{*+} \rightarrow D^0 \pi^+} / \Gamma_{D^{*+} \rightarrow \text{all}} &= (68.0 \pm 1.4 \pm 2.4)\% \\ \Gamma_{D^{*+} \rightarrow D^+ \pi^0} / \Gamma_{D^{*+} \rightarrow \text{all}} &= (31.0 \pm 0.4 \pm 1.6)\%.\end{aligned}$$

We are able to employ similar methods in the decays of the D^{*0} vector meson. In this instance, the possible PP decay channels are restricted by phase space to $D^{*0} \rightarrow D^0 \pi^0$. We calculate this width to be 27.2 ± 0.7 KeV, where we used $g_{D^{*0} D^0 \pi^0} = (0.1537 \pm 0.002)(2007.1)^{1/2} / \sqrt{2}$. To determine the radiative width $D^{*0} \rightarrow D^0 \gamma$ we apply relation (29) along with a small correction for the change in phase space to derive

$$\Gamma_{D^{*0} \rightarrow D^0 \gamma} \approx 3672 \times \Gamma_{D^{*+} \rightarrow D^+ \gamma} = (20.1 \pm 1.1) \text{ KeV}.$$

Thus the total D^{*0} width is (47.3 ± 1.3) KeV, although the result is sensitive to the supermultiplet prediction $g_{VVP}(D^*)/g_{VVP}(D^*) = 2/m_{D^*}$. Consequently, we predict the following branching fractions

$$\begin{aligned}\Gamma_{D^{*0} \rightarrow D^0 \pi^0} / \Gamma_{D^{*0} \rightarrow \text{all}} &\approx (57.5 \pm 2.2)\% \\ \Gamma_{D^{*0} \rightarrow D^0 \gamma} / \Gamma_{D^{*0} \rightarrow \text{all}} &\approx (42.5 \pm 2.6)\%\end{aligned}$$

which are in fair agreement with the CLEO data:

$$\begin{aligned}\Gamma_{D^{*0} \rightarrow D^0 \pi^0} / \Gamma_{D^{*0} \rightarrow \text{all}} &= (64 \pm 2.4 \pm 4.5)\% \\ \Gamma_{D^{*0} \rightarrow D^0 \gamma} / \Gamma_{D^{*0} \rightarrow \text{all}} &\approx (36 \pm 2.4 \pm 4.5)\%.\end{aligned}$$

6 Conclusions

This study of two-body meson decays has shown that the supermultiplet method unifies meson decays quite well, even for the light quarks. The most significant finding

is that the coupling between mesons is susceptible to symmetry breaking mechanisms, but in a *regular* way, allowing us to successfully extrapolate to decay rates for other processes. In particular, the methods are readily applicable to heavy quark examples, as highlighted by our examination of D^* processes.

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Table captions.

Table 1 Results for g_{VPP} determination.

Table 2 Results for g_{VVP} determination.

Figure captions.

Figure 1 Quark line or duality diagram.

Figure 2 Vector meson dominance.

Figure 3 Support for the supermultiplet symmetry condition $2g_{VPP} = m_1 g_{VVP}$ in the decays $\phi \rightarrow K^0 K^0$ and $\phi \rightarrow \rho\pi$.

Decay	Factor	g_{VPP}
$\rho^\pm \rightarrow \pi^\pm \pi^0$	$\sqrt{2}$	4.24 ± 0.05
$\rho^0 \rightarrow \pi^+ \pi^-$	$\sqrt{2}$	4.30 ± 0.03
$\rho^\pm \rightarrow \pi^\pm \eta$	$\sqrt{1/6}(\cos \theta_P + \sqrt{2} \sin \theta_P)$	$< 4.17 \pm 0.16$
$K^{*\pm} \rightarrow (K\pi)^\pm$	$1, \sqrt{1/2}$	4.59
$K^{*0} \rightarrow (K\pi)^0$	$\sqrt{1/2}, 1$	4.55
$\phi \rightarrow K^+ K^-$	$\sqrt{3/2} \cos \theta_V$	4.82 ± 0.05
$\phi \rightarrow K_L^0 K_S^0$	$\sqrt{3/2} \cos \theta_V$	4.99 ± 0.06
$D^{*+} \rightarrow D^0 \pi^+$	1	$< 10.2 \pm 1.0$
$D^{*+} \rightarrow D^+ \pi^0$	$1/\sqrt{2}$	$< 10.3 \pm 1.1$
Decay	Coupling factor	g'_{VPP}
$\rho^0 \rightarrow e^+ e^-$	$\sqrt{1/2}$	3.57 ± 0.09
$\rho^0 \rightarrow \mu^+ \mu^-$	$\sqrt{1/2}$	3.41 ± 0.11
$\omega \rightarrow e^+ e^-$	$\sqrt{1/6} \sin \theta_V$	4.40 ± 0.06
$\omega \rightarrow \mu^+ \mu^-$	$\sqrt{1/6} \sin \theta_V$	> 2.70
$\phi \rightarrow e^+ e^-$	$\sqrt{1/6} \cos \theta_V$	4.07 ± 0.05
$\phi \rightarrow \mu^+ \mu^-$	$\sqrt{1/6} \cos \theta_V$	4.46 ± 0.31
$J/\psi \rightarrow e^+ e^-$	$2/3$	7.55 ± 0.29
$J/\psi \rightarrow \mu^+ \mu^-$	$2/3$	7.72 ± 0.31
$\Upsilon \rightarrow e^+ e^-$	$1/3$	13.36 ± 0.53
$\Upsilon \rightarrow \mu^+ \mu^-$	$1/3$	13.47 ± 0.32
$\Upsilon \rightarrow \tau^+ \tau^-$	$1/3$	11.63 ± 0.72

$$\theta_V = 140.6^\circ, \theta_P = 10.5^\circ$$

Table 1.

Decay	Factor	g_{VVP} $\times 10^{-2} \text{ MeV}^{-1}$
$\phi \rightarrow \rho\pi$	$\sqrt{2/3}(\cos \theta_V + \sqrt{2} \sin \theta_V)$	1.062 ± 0.030
$\rho^\pm \rightarrow \pi^\pm \gamma$	$1/3$	0.923 ± 0.055
$\rho^0 \rightarrow \pi^0 \gamma$	$1/3$	1.216 ± 0.156
$\rho^0 \rightarrow \eta \gamma$	$(\cos \theta_P + \sqrt{2} \sin \theta_P)/\sqrt{3}$	0.984 ± 0.094
$\omega \rightarrow \pi^0 \gamma$	$(\cos \theta_V - \sin \theta_V)/\sqrt{3}$	0.878 ± 0.033
$\omega \rightarrow \eta \gamma$	$(\sqrt{2} \cos(\theta_V + \theta_P) + \sin \theta_V \cos \theta_P)/3$	0.919 ± 0.165
$\phi \rightarrow \pi^0 \gamma$	$(\cos \theta_V + \sqrt{2} \sin \theta_V)/\sqrt{3}$	0.731 ± 0.040
$\phi \rightarrow \eta \gamma$	$(\sqrt{2} \sin(\theta_V + \theta_P) - \cos \theta_V \cos \theta_P)/3$	0.603 ± 0.020
$\phi \rightarrow \eta' \gamma$	$(\sqrt{2} \cos(\theta_V + \theta_P) + \cos \theta_V \sin \theta_P)/3$	< 1.69
$K^{*\pm} \rightarrow K^\pm \gamma$	$1/3$	0.905 ± 0.045
$K^{*0} \rightarrow K^0 \gamma$	$-2/3$	0.733 ± 0.036
$J/\psi \rightarrow \eta_c \gamma$	$4/3$	0.308 ± 0.073
$\eta' \rightarrow \rho^0 \gamma$	$(\sqrt{2} \cos \theta_P - \sin \theta_P)/\sqrt{3}$	0.693 ± 0.041
$\eta' \rightarrow \omega \gamma$	$-(\sin \theta_P \sin \theta_V + \sqrt{2} \sin(\theta_P + \theta_V))/3$	0.698 ± 0.051
$\pi^0 \rightarrow \gamma \gamma$	$\sqrt{2}/3$	0.915 ± 0.053
$\eta \rightarrow \gamma \gamma$	$\sqrt{2/3}(\sqrt{2} \sin \theta_P + \cos \theta_P)/3$	0.870 ± 0.056
$\eta' \rightarrow \gamma \gamma$	$\sqrt{2/3}(\sqrt{2} \cos \theta_P - \sin \theta_P)/3$	0.730 ± 0.055
$\eta_c \rightarrow \gamma \gamma$	$8/9$	0.484 ± 0.337

$$\theta_V = 140.6^\circ, \theta_P = 10.5^\circ$$

Table 2.

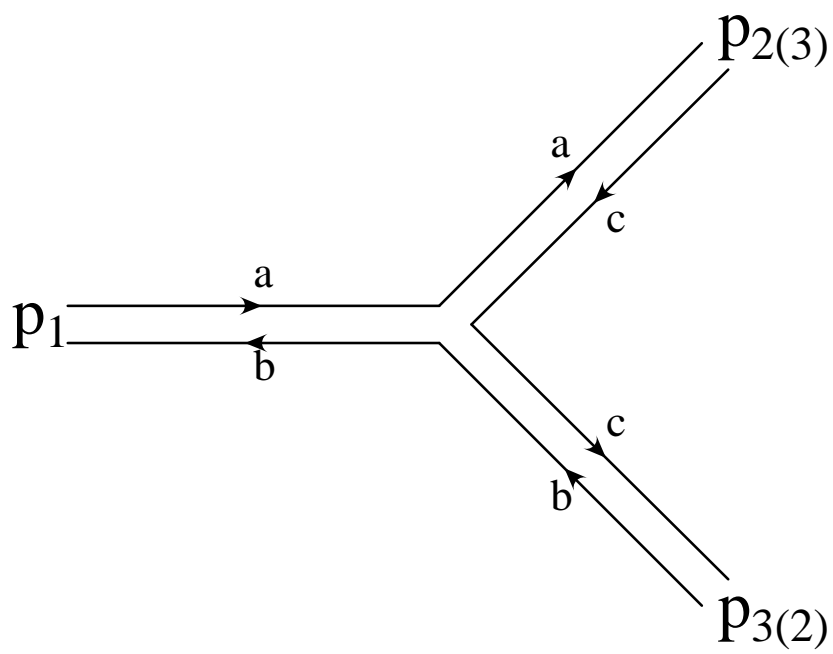


Figure 1.

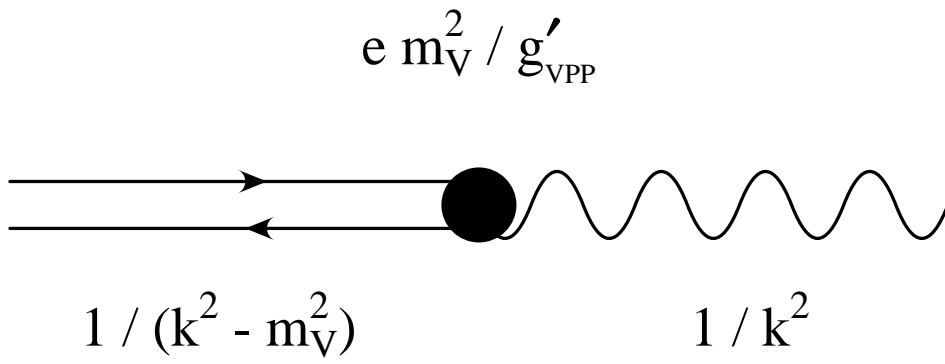


Figure 2.

$$m_1 g_{VVP} - 2g_{VPP}$$

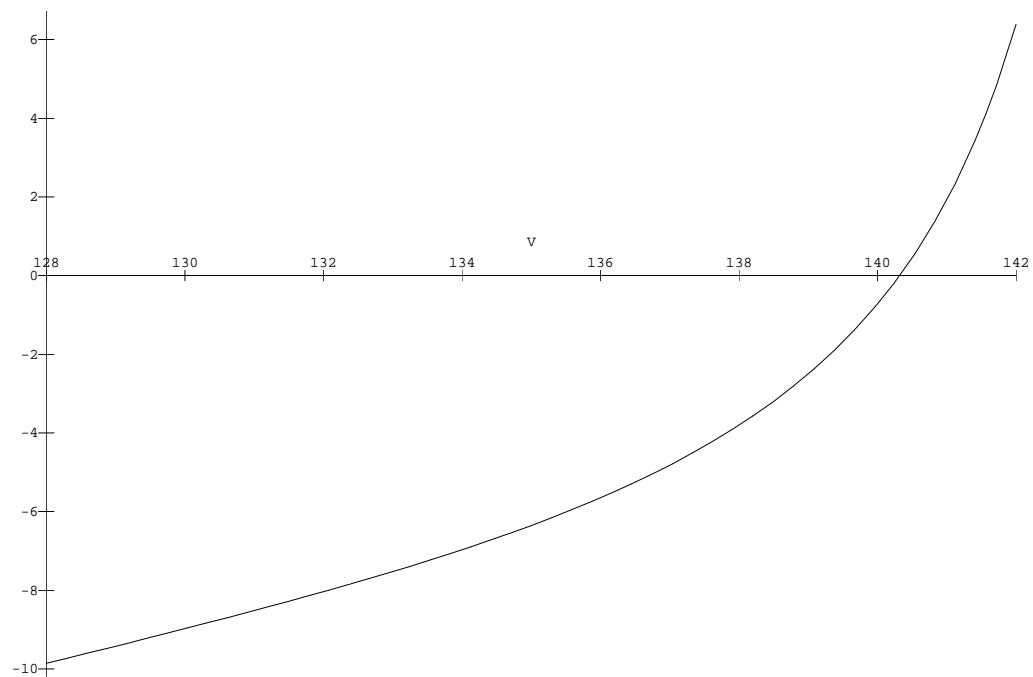


Figure 3.